Active Search for Sparse Signals with Region Sensing

Yifei Ma* and Roman Garnett** and Jeff Schneider*

*Carnegie Mellon University **Washington University in St. Louis

\[ \text{yifeim@cs.cmu.edu} \]

** Motivation **

Region sensing (aggregate value)

Task: localize the sources

Challenge: change altitude to balance coverage and fidelity

\[ \begin{align*}
\text{Algorithm} & \\
\text{Sample complexity} & \\
\text{Real satellite images}
\end{align*} \]

** Measurement Model **

Discretize search space to \( n \) grid points (e.g., 1d search)

Signal vector

\[ \mathbb{R}^n \ni \beta = \] Signal vector

Region choice

\[ \mathbb{R}^n \ni x_t = \] Region sensing

\( \epsilon \)-value

\( N(0,1) \)

Sensing outcome at step \( t \) is \( y_t = x_t^\top \beta + \epsilon_t \), where \( \epsilon_t \sim N(0,1) \).

** Objective **

\( \text{Choose } X = \{ x_t \}_{t=1}^{T} \text{ to discover } S \text{ (let } \hat{S}_T \text{ be the estimate).} \)

\( \text{Loss is } d(S, \hat{S}_T) = \frac{1}{2} |S \triangle \hat{S}_T|, \] where \( \triangle \) is the symmetric difference of two sets, \( S \triangle \hat{S}_T = (S \setminus \hat{S}_T) \cup (\hat{S}_T \setminus S) \).

** Proposed Algorithm: Region Sensing Index (RSI) **

\begin{itemize}
  \item[Require:] \( n, k, \mu \)
  \item[use uniform prior \( \pi(\beta) \) \] (1)
  \item[for \( t = 1, \ldots, T \) do]
  \item[pick \( x_t = \arg \max_{x \in X} I(\beta, y_t | x, \pi_{t-1}) \)] (2)
  \item[observe \( y_t = x_t^\top \beta + \epsilon_t \)]
  \item[update \( \pi_t(\beta) \propto \pi_{t-1}(\beta)p(y_t | \beta, x_{t-1}) \)] (3)
  \item[Ensure:] \( \hat{S}_T = \arg \min_{\beta} \mathbb{E} \left[ d(S, \hat{S}_T) \right] \) \( \beta \sim \pi_T \)
\end{itemize}

** Theoretical Guarantees in 1D Search **

<table>
<thead>
<tr>
<th>Design type</th>
<th>Sensing Algorithm</th>
<th>Bayes prior</th>
<th>Min ( T ) to get ( \epsilon )-risk</th>
<th>Sample complexity(^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>passive</td>
<td>Point sensing</td>
<td>( \pi_0 ) (( \mu \to \infty ))</td>
<td>( T \geq \frac{1}{2} \left( 1 - \frac{\epsilon}{\mu} \right) )</td>
<td>( \Theta(n) )</td>
</tr>
<tr>
<td>no</td>
<td>(any)</td>
<td>( \pi_0 ) (( \mu \to \infty ))</td>
<td>( T \leq \frac{1}{2} \left( 1 - \frac{\epsilon}{\mu} \right) )</td>
<td>( \Omega(\frac{n}{\epsilon^2}) )</td>
</tr>
<tr>
<td>active</td>
<td>RSI (ours)</td>
<td>( \pi_0 ) ( \max \text{ risk} )</td>
<td>( T \leq \frac{50}{\epsilon^2} \left( \log \left( \frac{2k}{\epsilon^2} \right) + 2k \log \left( \frac{2k}{\epsilon^2} \right) \right) )</td>
<td>( \tilde{O}(\frac{n}{\epsilon^2} + k) )</td>
</tr>
</tbody>
</table>

** Simulation Studies **

Search space is 1d; discretized to \( n = 1024 \) points.

\( \text{Signal is } 1\text{-sparse, with strength } \mu = 10. \)

\( \text{RSI: most efficient choice of measurements.} \)

\( \times \text{ CASS [1]: less efficient in general.} \)

\( \text{not anytime; produces only turning points.} \)

\( \times \text{ CS [2]: unconstrained; still less efficient.} \)

\( \text{Point: best passive constrained sensing.} \)

\( \text{To fully represent CASS, we show different choices of } \} \text{ given a priori (including the true value).} \)

** Real-World Datasets **

** Discussion and Related Work **

** Method / Paper **

<table>
<thead>
<tr>
<th>Region Sparse</th>
<th>Active sensing methods</th>
<th>Anytime search</th>
<th>Robust to non-iid noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bayesian optimization [4]</td>
<td>× × ✓</td>
<td>✓ ✓ ✓ ✓ ✓</td>
<td></td>
</tr>
<tr>
<td>Compressive sensing [2]</td>
<td>× × ✓ × × ✓</td>
<td>✓ ✓ ✓ ✓ ✓</td>
<td></td>
</tr>
<tr>
<td>CASS [1]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>RSI (ours)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

\(^*\) Compressive sensing requires unconstrained sensing that does not incorporate region constraints.

\(*\) “No active method (e.g. [1]) can fundamentally improve sample efficiency beyond log-periodic factors” \[3\]. This is only true in the case of unconstrained sensing.

\(1\) CASS is a branch-and-bound algorithm that produces results only near the end.

\(2\) CASS requires repetitive measurements on the same region to control branching error, which is not practical in static environments.

** References **


